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The Formula Method to do subtraction of mixed numbers

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Abstract

Subtraction of fractions involving mixed numbers confuses many pupils because of the many forms of the questions and the many alternative methods of performing the algorithm. The researcher created three formulae to do subtraction of mixed numbers. This study is a priori study to determine the effectiveness of using the three formulae which is called the "Formula Method" in doing subtraction of mixed numbers. The study involved five Year 5 Malaysian Primary school pupils in an urban primary school who were selected from 20 students who sat for a test on subtraction of mixed numbers. The participants for the study were pupils who demonstrated difficulties doing subtraction of mixed numbers.
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***Keywords*:** Mixed numbers; Alternative method; Formula Method; Year 5; Urban primary school; Difficulties

1. Introduction

Mathematics Education demands acquisition of basic mathematical concepts and skills. Among the major goals of Mathematics Education in the primary school is the acquisition of the knowledge of fractions (Nik Pa, 1989). Although the knowledge of fractions is regarded as a major goal of Mathematics Education, many pupils experience difficulties with fractions (Charles and Nason, 2000).

Being aware of the importance of the knowledge of fractions and difficulties faced by children in learning fractions, the researcher looked up books and articles to obtain guidance in preparing the lessons on fractions (Grossnickle and Brueckner, 1963; Chapin and Johnson, 2000; Miller, 2009; Booker et.al, 2004). An article by Peck and Connell (1991) claimed that physical materials must be used as the foundation to construct children's symbolic meaning and definitions of fractions. The researcher was greatly impressed by their paper titled "Using Physical Materials to Develop Mathematical Intuition in Fraction Part-Whole Situations", and hence employed the use of the fraction kit to teach subtraction of fractions involving mixed numbers.

Unfortunately, many of the researcher's pupils got confused and were unable to benefit from the use of the fraction kit. Hence the researcher decided to try a drill and practice method to teach subtraction of fractions involving mixed number. For the drill and practice method the algorithm was explained without giving any meaning to it and exercises were given for practice. However, the pupils were confused as to which method to use for a given

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problem. This is because of the diversity of questions and methods to do subtraction of fractions involving mixed numbers.

If a mixed number subtraction is represented as $A\frac{b}{c} - D\frac{e}{f}$, then the questions that are possible are as follows:

1. $A - D\frac{e}{f}$
2. $A\frac{b}{c} - D$
3. $A\frac{b}{c} - D\frac{e}{f}$, *where* $A > D, A > 0, D \geq 0$:
   a. *When* $b > e$ *and* $c = f$
   b. *When* $e > b$ *and* $c = f$
   c. *When* $b > e$ *and* $c \neq f$
   d. *When* $e > b$ *and* $c \neq f$

There are 3 methods of doing subtraction of mixed number depending on the question.

Method 1 **: Convert to improper fractions**

- Can be used for any type of question involving mixed numbers
- Example,

$$2\frac{3}{5} - 1\frac{4}{5}$$
$$= \frac{13}{5} - \frac{9}{5}$$
$$= \frac{4}{5}$$

Method 2 **: Partially converting to improper fraction**

- Can be used for type 1, 2 and 3(b) only
- Example,

$$6 - 3\frac{1}{4}$$
$$= 5\frac{4}{4} - 3\frac{1}{4}$$
$$= 2\frac{3}{4}$$

Method 3 **: Without converting to improper fractions**

- Can be used for type 2 and 3(a) only
- Example,

$$7\frac{5}{8} - 4$$
$$= 3\frac{5}{8}$$

Hence the diversity involved in dealing with mixed number subtraction led to confusion among my pupils when doing subtraction of fractions involving mixed numbers.

Surfing the internet for other methods to do subtraction of fractions involving mixed numbers lead to an article by Z-Math (2010). This article provided the following general instruction to subtract any fraction from any whole number: first decrease the whole number by 1 and then subtract the numerator from the denominator of the proper fraction given and write this difference as the new numerator over the same denominator. The formula given was,

$$A - \frac{B}{C} = (A-1) + \frac{(C-B)}{C}$$

This formula can be modified and used for subtraction of a mixed number from a whole number as follows:

$$\begin{aligned} A - D\frac{e}{f} = A - \frac{e}{f} - D &= (A-1) + \frac{f}{f} - \frac{e}{f} - D \\ &= (A-1) + \frac{(f-e)}{f} - D \\ &= (A-D-1) + \frac{(f-e)}{f} \end{aligned}$$

So the researcher then decided that if the question involved two mixed numbers, the second formula would be obtained as shown below.

$$\begin{aligned} A\frac{b}{c} - D\frac{e}{f} = A + \frac{b}{c} - D - \frac{e}{f} &= A - \frac{e}{f} - D + \frac{b}{c} \\ &= (A-D-1) + \frac{(f-e)}{f} + \frac{b}{c} \end{aligned}$$

The third formula is for $A\frac{b}{c} - D$ which merely requires subtracting the whole numbers $A - D$ and adding the fraction part.

$$A\frac{b}{c} - D = (A-D) + \frac{b}{c}$$

Hence, the Formula Method was created for the three types of questions by the researcher.

Table 1: Formula Method

Type of questions	$A\frac{b}{c} - D$, where A>0 and D>0	$A - D\frac{e}{f}$, where A>0 and D≥0	$A\frac{b}{c} - D\frac{e}{f}$, where A>0 and D≥0
Formula	$(A-D) + \frac{b}{c}$	$(A-D-1) + \frac{(f-e)}{f}$	$(A-D-1) + \frac{(f-e)}{f} + \frac{b}{c}$

The main focus of this study is to describe the effects of using the Formula Method on pupils' performance in doing subtraction of fractions involving mixed numbers.

2. Methodology

Five Year 5 pupils (all 11 years old) from an urban primary school in Malaysia were participants of this study. They were selected from 20 pupils who sat for the pre-test. A time frame was not given for the pre-test. The pre-test consists of three sections, mixed number minus whole number, whole number minus mixed number and mixed number minus mixed number. The pre-test was followed by a group interview session.

The next stage was the two one hour teaching and learning sessions to familiarize the pupils with the Formula Method. The pupils were required to recognise the structure of the question and relate it to the respective formulae.

Each teaching and learning session included an exercise for evaluation which was to be completed within the session. The exercise questions were then discussed by the pupils showing their working on the board.

Finally the post-test was administered. Again no time frame was attached to do the test. The post-test was followed by a group interview. The post-test is the pre-test. Field notes of observable behaviour were recorded for every teaching and learning session, pre-test and post-test. Interviews were audio taped. All test papers and exercise work were collected. A reflective journal was kept of every interaction with the pupils.

3. Findings

During the pre-test pupils answered hurriedly. Three out of the five pupils could do questions 1 and 2 of the form $A\frac{b}{c} - D = (A - D) + \frac{b}{c}$ which merely required subtracting the whole number part (See Figure 1)

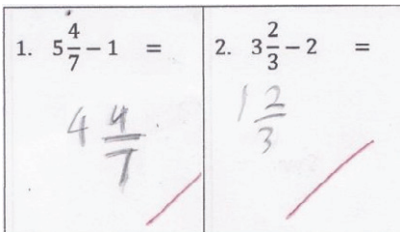

Figure 1: Questions 1 and 2 done correctly

This led many to use the algorithm of questions 1 and 2 to do questions 3 and 4 of the form $A - D\frac{e}{f}$ (see Figure 2).

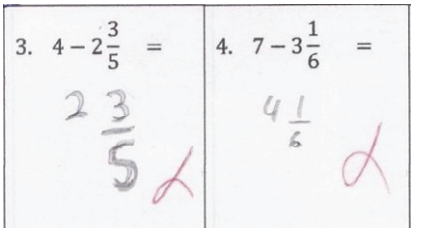

Figure 2: Questions 3 and 4 done using the algorithm of questions 1 and 2

Questions 5 and 6 of the form, $A\frac{b}{c} - D\frac{e}{f}$, *where* $A > D$, $b>e$, and $c = f$, were done correctly by four out of five of the pupils (but they only did not simplify the answer). This was because it merely involves subtracting the whole numbers and the numerators from left to right (see Figure 3)

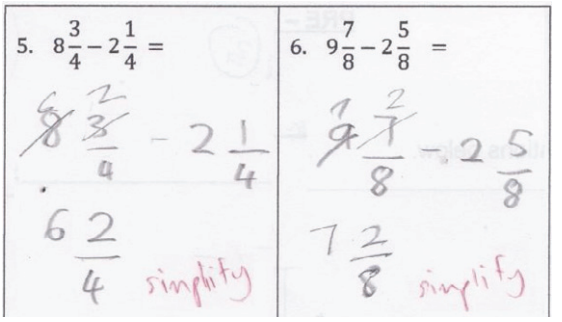

Figure 3: Questions 5 and 6 done correctly but answer has not been simplified

Questions 7 and 8 were done using the algorithm of questions 5 and 6 (see Figure 4).

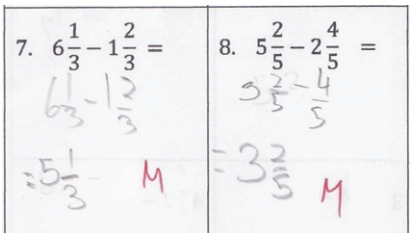

Figure 4: Questions 7 and 8 done using the algorithm of questions 5 and 6

Questions 9, 10, 11 and 12 were left unanswered or they just did subtraction of all possible values (see Figure 5)

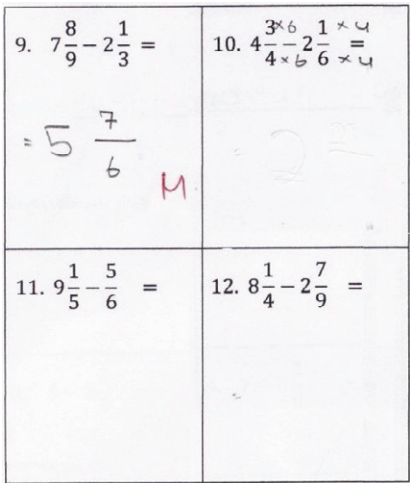

Figure 5: Question 9, the whole numbers, numerators and denominators subtracted to obtain the answer shown and questions 10, 11 and 12 are left unanswered

Hence the scores for the pre-test were basically from questions 1, 2, 5 and 6. Of the three types of questions subtraction of a whole number from a mixed number was done well (questions 1 and 2).

The interview after the pre-test revealed that the pupils did not know which algorithm to use to do the different questions which appear to be alike. They obviously did not understand what they were doing and they could not even do the questions using a mechanical method because they were unable to recognise the structure of the question and relate it to a specific algorithm.

After the interview session, the next day, the teaching and learning session began. The teaching and learning sessions at first shocked the pupils when they saw the formulae. When the formulae were explained, they were still looking confused. Then examples were given and they began to understand. When Exercise 1 was served to them, there still was some confusion about the formulae to use and pupils kept comparing each others work and also requested affirmations from the teacher. During the second teaching and learning session they were more comfortable with the formula method and were able to match questions to formula. Exercise 2 was done more confidently and independently. They checked each others' work and could identify each others' errors. The session went well.

The post test was administered the next day. The data for the pre-test and post test are shown in Table 2. The data has been analysed based on the type of questions: section A: mixed number minus whole number, section B: whole number minus mixed number and section C: mixed number minus mixed number.

Table 2: Pre-test and post-test scores

Pupil	Section A		Section B		Section C		Overall	
	Pre-test	Post-test	Pre-test	Post-test	Pre-test	Post-test	Pre-test	Post-test
Pupil 1	0	100	50	100	25	100	25	100
Pupil 2	0	100	0	100	0	75	0	83
Pupil 3	100	50	0	100	0	38	17	50
Pupil 4	100	100	0	100	0	88	17	92
Pupil 5	100	100	0	100	0	75	17	83
Mean	60	90	10	100	5	75	15	82

Every section shows an increase in score from the pre-test to the post-test. The overall increase is 67% from pre-test to post-test. Section A has a 30% increase, Section B has a 90% increase and Section C has a 70% increase.

The scores for four out of five participants for Section A, mixed number minus the whole number are 100% in the post test. There were only two questions in this section. Although the overall pre-test score for this section is less than the overall post-test scores, in the pre-test three pupils scored 100% in this section. Performance of pupils in this section for the pre-test was the best compared to the other two sections. This is because the algorithm merely involves subtracting the whole numbers and copying the fraction part to obtain the answer. The expected post-test score for all was 100% in this section because it uses the simplest formula in the Formula method. However, the pupil who got one question wrong used the correct formula but made a careless mistake copying the fraction (see Figure 6).

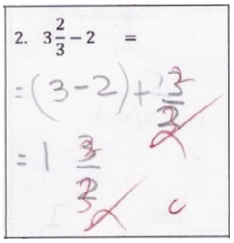

Figure 6: Careless mistake when doing a Section A question

The scores for every participant in Section B, whole number minus mixed number are 100% in the post-test. There were only two questions in this section. The formula involved to do the questions is a very simple formula. Hence all the pupils were able to do both the questions correctly. The scores of pupils in the pre-test however were zero and 50%. This was because pupils had not recognised the questions in Section B to be different from those in Section A and hence applied the Section A algorithm to do Section B questions. However, one pupil who scored 50% (1 correct answer) for Section B in the pre-test used the Section B algorithm for Section A questions and got both Section A questions wrong (see Figure 7) but for Section B question got one correct answer but made a careless mistake in subtraction for the other question (see Figure 8). This illustrates the confusion of pupils before the Formula method was employed.

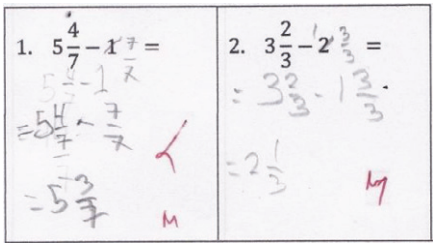

Figure 7: Section A question done with a Section B algorithm in the pre-test

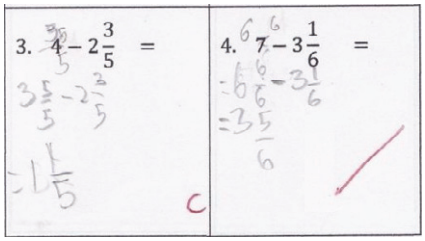

Figure 8: Careless mistake in subtraction when doing a Section B question in the pre-test

Another reason for Section A and Section B questions to be done easily using the Formula method is because pre-requisites such as recall of basic multiplication facts and finding common denominators were not required.

In Section C Pupil 1 scored 100%. Pupils 2, 4 and 5 scored 75% or more. They had made several careless mistakes (see Figure 9). This may be because of the many steps involved when using the formula when two mixed numbers are involved.

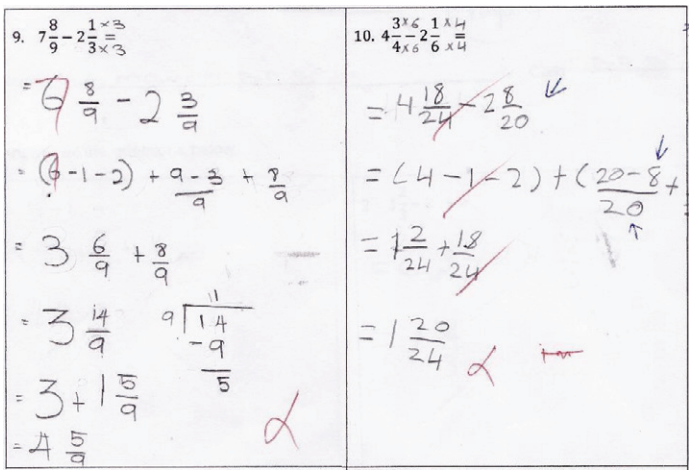

Figure 9: Careless mistakes in doing Section C questions

The same formula is also applied when a proper fraction is subtracted from a mixed number as in question 11(see Figure 10).

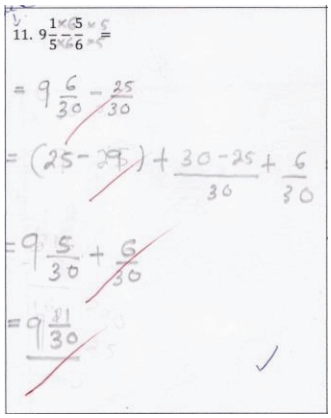

Figure 10: Question 11, mixed number minus a proper fraction

Pupil 3 however, scored only 38% for section C. Examining his paper revealed that he was able to use the Formula Method but he may have had problems recalling his times tables or became tired. He knew how to use the Formula Method and he used it for the first four questions correctly. For the next four questions he just mixed up the steps. When he was asked the next day, why he had not done the other four questions properly, he just looked down and did not respond.

However, during the post-test all the pupils appeared cheerful and confident. All five were concentrating on the test. The interview after the post-test revealed that they were confident to use the formula method (even Pupil 3). Their work reflected their confidence in using the Formula method.

4. Conclusion

The Formula method has been effective to reduce confusion for pupils in this study to do subtraction of fractions involving mixed numbers. The study also revealed that related knowledge such as multiplication facts were required when doing mixed number minus mixed number and mixed number minus proper fraction questions. Other related mathematical knowledge are simplifying fractions and converting improper fractions to proper fractions. The lack of knowledge of these other pre-requisites to do subtraction of fractions involving mixed numbers cannot be overcome by using the Formula method. The Formula method only enables pupils to recognise the structure of the question and select the correct formula which then leads the pupil to the correct algorithm.

The pupils did not understand each of the formulae nor did they understand why a particular formula leads to obtain the correct answer. Hence although the Formula Method served to reduce confusion from the diversity of the questions and enabled pupils in this study to obtain the correct answer, the pupils answered the questions using a meaningless algorithm.

However, the three formulae in the Formula method can be explained meaningfully to the pupils. The concepts involved in creating the formulae in the Formula Method are within the Malaysian Primary school curriculum and hence are within the capacity of understanding of the pupils. Hence the next phase of the research will involve explaining the three formulae and then ask pupils to work out the answers for given problems and then ask them to explain by drawing or otherwise to show what the algorithm actually represents.

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